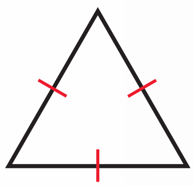
Geometry Date_____ 4.1 Notes
Triangles and Angles (pp 194–197)

NAMES OF TRIANGLES

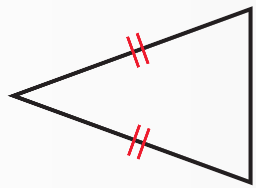
Classification by Sides

EQUILATERAL TRIANGLE



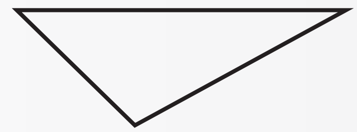
3 congruent sides

ISOSCELES TRIANGLE



At least 2 congruent sides

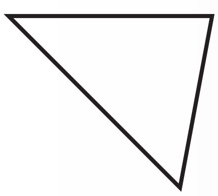
SCALENE TRIANGLE



No congruent sides

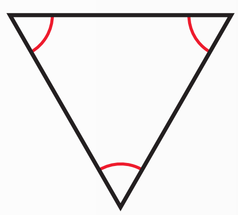
Classification by Angles

ACUTE TRIANGLE



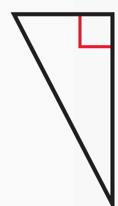
3 acute angles

EQUIANGULAR TRIANGLE



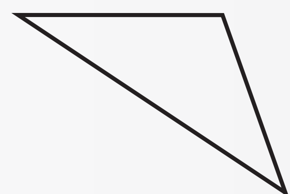
3 congruent angles

RIGHT TRIANGLE



1 right angle

OBTUSE TRIANGLE

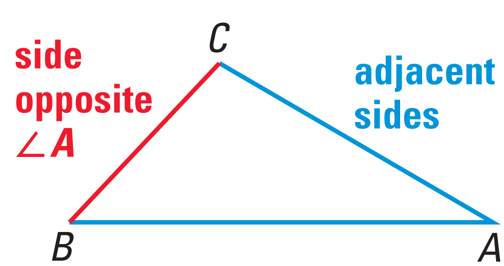


1 obtuse angle

Note: An equiangular triangle is also acute.

Each of the three points joining the sides of a triangle is a **vertex**. (The plural of vertex is *vertices*.) For example, in $\triangle ABC$, points A, B, and C are vertices.

In a triangle, two sides sharing a common vertex are **adjacent sides**. In $\triangle ABC$, $\overline{CA}$ and $\overline{BA}$ are adjacent sides. The third side, $\overline{BC}$, is the side *opposite* $\angle A$.



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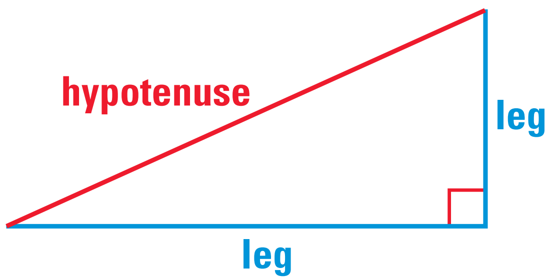
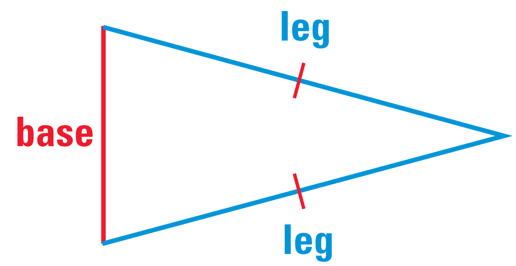
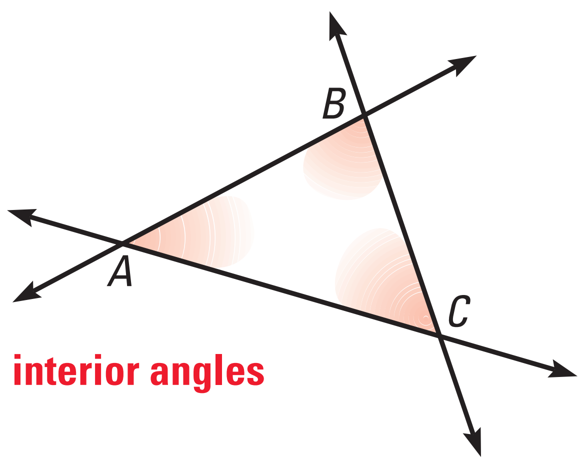
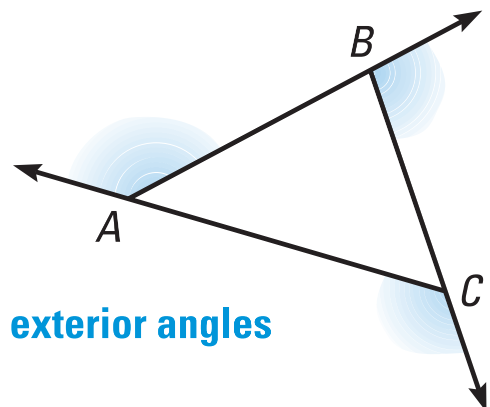


Right triangle



Isosceles triangle



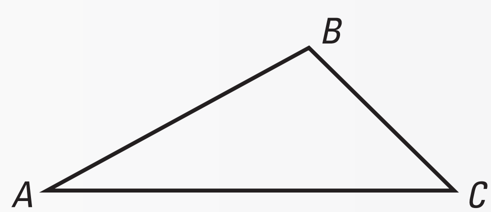


THEOREM

THEOREM 4.1 *Triangle Sum Theorem*

The sum of the measures of the interior angles of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

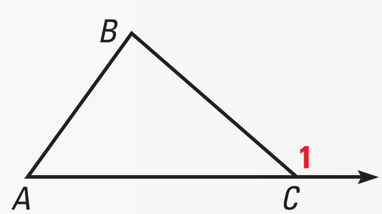


THEOREM

THEOREM 4.2 *Exterior Angle Theorem*

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$



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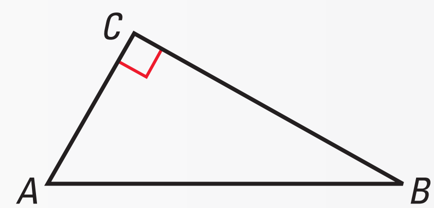
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

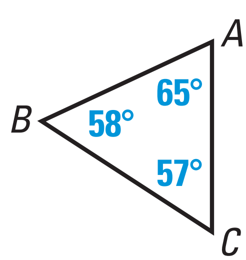
$m\angle A + m\angle B = 90°$



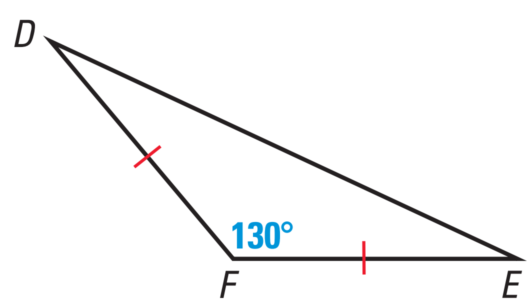
EXAMPLE 1 *Classifying Triangles*

When you classify a triangle, you need to be as specific as possible.

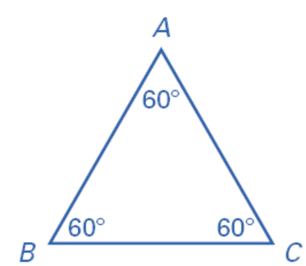
a. $\triangle ABC$ has three acute angles and no congruent sides. It is an acute scalene triangle. ($\triangle ABC$ is read as "triangle ABC.")



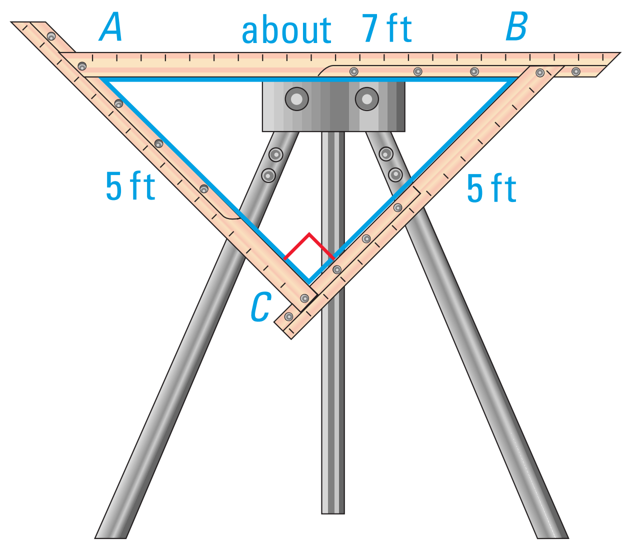
b. $\triangle DEF$ has one obtuse angle and two congruent sides. It is an obtuse isosceles triangle.



6. Classify the triangle.



EXAMPLE 2 Identifying Parts of an Isosceles Right Triangle



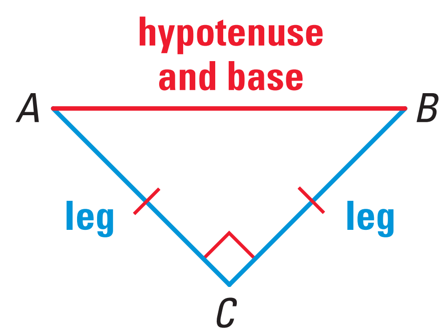
The diagram shows a triangular loom.

a. Explain why $\triangle ABC$ is an isosceles right triangle.

b. Identify the legs and the hypotenuse of $\triangle ABC$. Which side is the base of the triangle?

Solution

a. In the diagram, you are given that $\angle C$ is a right angle. By definition, $\triangle ABC$ is a right triangle. Because $AC = 5$ ft and $BC = 5$ ft, $\overline{AC} \cong \overline{BC}$. By definition, $\triangle ABC$ is also an isosceles triangle.



b. Sides $\overline{AC}$ and $\overline{BC}$ are adjacent to the right angle, so they are the legs. Side $\overline{AB}$ is opposite the right angle, so it is the hypotenuse. Because $\overline{AC} \cong \overline{BC}$, side $\overline{AB}$ is also the base.

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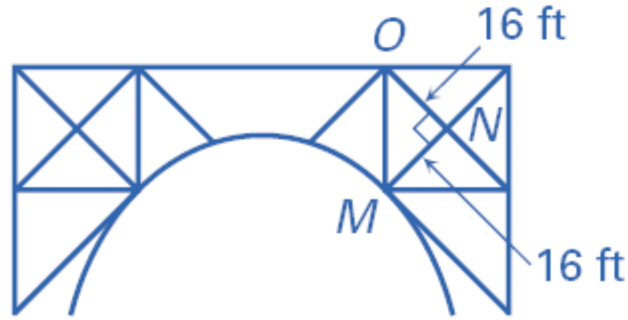
The diagram shows a bridge.

7. Explain why $\triangle MNO$ is an isosceles right triangle.



8. Identify the legs and hypotenuse of $\triangle MNO$. Which side is the base of the triangle?

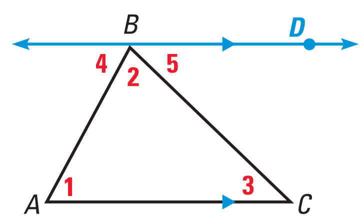
To prove some theorems, you may need to add a line, a segment, or a ray to the given diagram. Such an *auxiliary line* is used to prove the Triangle Sum Theorem.

Proof

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Plan for Proof By the Parallel Postulate, you can draw an auxiliary line through point B and parallel to $\overline{AC}$. Because $\angle 4$, $\angle 2$, and $\angle 5$ form a straight angle, the sum of their measures is $180°$. You also know that $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$ by the Alternate Interior Angles Theorem.



Statements	Reasons
1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate
2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	**2.** Angle Addition Postulate and definition of straight angle
3. $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem
4. $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**5.** Substitution property of equality

STUDENT HELP

Study Tip

An auxiliary line, segment, or ray used in a proof must be justified with a reason.

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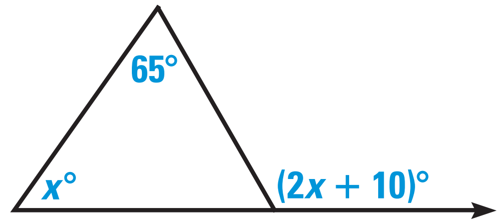
EXAMPLE 3 *Finding an Angle Measure*

You can apply the Exterior Angle Theorem to find the measure of the exterior angle shown. First write and solve an equation to find the value of x:

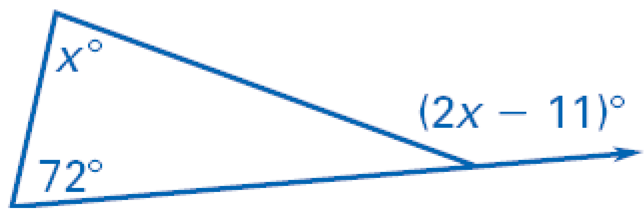


$x° + 65° = (2x + 10)°$ **Apply the Exterior Angles Theorem.**

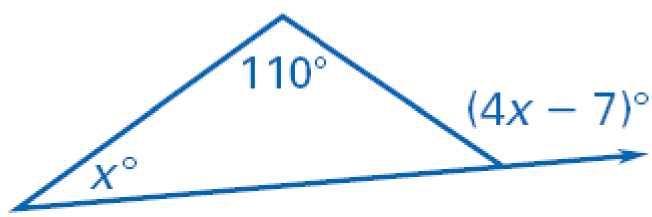
$55 = x$ **Solve for *x*.**

▶ So, the measure of the exterior angle is $(2 \cdot 55 + 10)°$, or $120°$.

9. Find the value of x. Then find the measure of the exterior angle.



10. Find the value of x. Then find the measure of the exterior angle.



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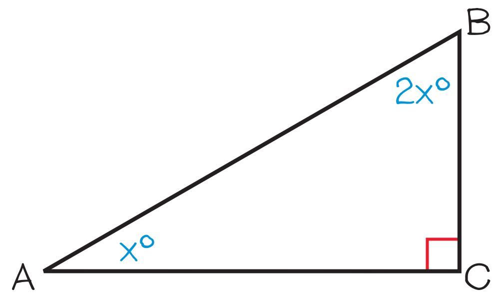
EXAMPLE 4 *Finding Angle Measures*

The measure of one acute angle of a right triangle is two times the measure of the other acute angle. Find the measure of each acute angle.

SOLUTION

Make a sketch. Let $x° = m\angle A$.
Then $m\angle B = 2x°$.



$x° + 2x° = 90°$ **The acute angles of a right triangle are complementary.**

$x = 30$ **Solve for *x*.**

▶ So, $m\angle A = 30°$ and $m\angle B = 2(30°) = 60°$.

11. The measure of one acute angle of a right triangle is one fourth the measure of the other acute angle. Find the measure of each acute angle.

12. The measure of one acute angle of a right triangle is five times the measure of the other acute angle. Find the measure of each acute angle.

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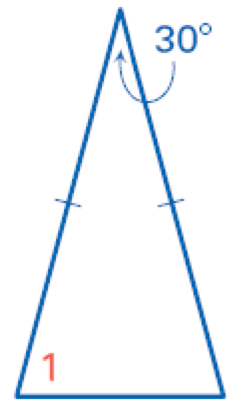

13. _____ Find $m\measuredangle 1$.

A. 30°

B. 60°

C. 75°

D. 150°

E. You need more information to solve this problem.

14. Sketch an obtuse scalene triangle. Label its interior angles 1, 2, and 3. Then draw its exterior angles. Shade the exterior angles.